7. Noncompliance and Instrumental Variables

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Where are we? Where are we going?

- We’ve covered randomized experiments (no confounding)
- We’ve covered selection on observables (no unmeasured confounding)
- What if there is unmeasured confounding? What can we do?

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- We've covered selection on observables (no unmeasured confounding)
- What if there is unmeasured confounding? What can we do?
- First approach we will explore: **instrumental variables**
  - First: motivate IV through experiments and noncompliance.
  - Next: discuss how it relates to classical econometric methods like 2SLS?

Source: Causal Inference for the Brave and True By Matheus Facure Alves

1/ Randomized Experiments with Noncompliance

Noncompliance

- Get-out-to-vote (GOTV) experiment with door-to-door canvassing:

Source: https://www.dreamstime.com/political-meeting-candidates-pre-election-campaign-concept-multinational-people-support-banners-man-woman-voters-flat-image185175984

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- Households are randomized so treatment assignment is unconfounded
  - $Z_i = 1$ for assigned to treatment (canvassing attempted),
  - $Z_i = 0$ for assigned to control (no canvassing attempted).
- **Noncompliance**: units don't follow treatment assignment.
  - Units assigned to treatment take control or vice versa.
  - $D_i = 1$ for actually took treatment (heard canvasser message).
  - $D_i = 0$ for actually took control (didn't answer the door).
  - Full compliance means $Z_i = D_i$ for all i.

How to Handle Noncompliance?

- Two approaches commonly seen in applied studies:

  1. **Intent-to-treat** (ITT) analysis: just use randomization.
     - Use Z_i as the treatment and analyze as a typical experiment.
     - Downside: can't learn about the effect of actually being canvassed!
  2. **As-treated** analysis: just use treatment uptake.
     - As if D_i was randomly assigned.
     - Not valid if uptake is **correlated** with outcome.
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- Alternative: leverage latent strata of **compliance types**.

Setup

- Treatment assignment, $Z_i \in \{0, 1\}$; treatment uptake, $D_i \in \{0, 1\}$,
- Treatment uptake is affected by assignment: $D_i(z)$
    - $D_i(1) = 1$ if assigned to canvassing, I **would** open my door.
    - $D_i(1) = 0$ if assigned to canvassing, I **would not** open my door.
    - Noncompliance means $D_i(z) \neq z$ for some i.
- Consistency for the observed treatment as usual:

$$D_i = D_i(Z_i) = Z_i D_i(1) + (1 - Z_i) D_i(0)$$

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- Canvassing is an example of **one-sided noncompliance**.
  - People might refuse treatment when offered (i.e., $D_i(1) = 0$)
  - But no one receives treatment if in control (i.e., $D_i(0) = 0, \ \forall \ i$)
  - **Two-sided noncompliance** is when you can refuse to comply with treatment **or** control.

Potential Outcomes

- Outcomes now dependant on assignment and uptake: $Y_i(z, d)$.
  - $Y_i(1, 1)$: would I vote if I were assigned to canvassing and received it?

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- Can only observe two potential outcomes: $Y_i(1, D_i(1))$ and $Y_i(0, D_i(0))$
    - $Y_i(1, D_i(1))$: potential outcome when assigned canvassing and whether uptake occurs for unit i when assigned to canvassing.
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  - $Y_i(1, 1 - D_i(1))$: not possible to ever observe! (cross-world or counterfactual)
- Consistency assumption: $Y_i = Y_i(Z_i, D_i(Z_i))$

Some Notations

- Let's use 0/1 subscripts for assignment, and t/c subscripts for uptake:

$$n_1 = \sum_{i=1}^{n} Z_i \quad n_0 = \sum_{i=1}^{n} (1 - Z_i) \quad n_t = \sum_{i=1}^{n} D_i \quad n_c = \sum_{i=1}^{n} (1 - D_i)$$

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- Average outcomes and uptake in each assignment group:

$$\overline{Y}_1 = \frac{1}{n_1} \sum_{i=1}^{n} Z_i Y_i \quad \overline{Y}_0 = \frac{1}{n_0} \sum_{i=1}^{n} (1 - Z_i) Y_i$$

$$\overline{D}_1 = \frac{1}{n_t} \sum_{i=1}^{n} Z_i D_i \quad \overline{D}_0 = \frac{1}{n_c} \sum_{i=1}^{n} (1 - Z_i) D_i$$

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- Assumption 1: **randomization** $[\{Y_i(d, z), \forall d, z\}, D_i(1), D_i(0)] \perp\!\!\!\perp Z_i$
  - For observational uses of IV, might condition on some $\mathbf{X}_i$.

Defining ITT Effects

- **Intent-to-treat** (ITT) effects are just the ATEs of Z_i:

$$\text{ITT}_D = \frac{1}{n}\sum_{i=1}^{n} D_i(1) - D_i(0) \qquad \text{ITT}_Y = \frac{1}{n}\sum_{i=1}^{n} Y_i(1, D_i(1)) - Y_i(0, D_i(0))$$

  - SATE of assignment on treatment uptake & the outcome.
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- Under randomization of Z_i, everything just like Neyman approach.
  - Variances, tests, CIs all standard.

- Problem: ITT_Y is a combination of true effect of D_i and noncompliance
  - Effect of D_i may be more externally valid than Z_i.

2/ Compliance Types

Compliance Status

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- Compliance is a function of potential outcomes so it is **pretreatment**!
    - $\rightsquigarrow$ treatment assignment independent of C_i

ITTs Among the Compliance Groups

- Compliance type indicator, $C_i \in \{\text{co}, \ \text{nc}\}$:
  - Number of compliers: $n_{\text{co}} = \sum_{i=1} \mathbf{1}(C_i = \text{co})$.
  - Proportion of compliers: $\pi_{\text{co}} = n_{\text{co}}/n$.
  - Same for noncompliers: n_{nc} and π_{nc}.

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- ITT on uptake is directly related to compliance type:

$$\text{ITT}_{D,\text{nc}} = \frac{1}{n_{\text{nc}}} \sum_{i:C_i=\text{nc}} D_i(1) - D_i(0) = 0$$

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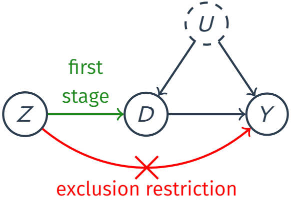
  - Intuition: no effect of assignment on uptake for noncompliers!
- Implies that overall ITT on uptake equals the **proportion of compliers**:

$$\text{ITT}_D = \pi_{\text{co}} \cdot \text{ITT}_{D,\text{co}} + \pi_{\text{nc}} \cdot \text{ITT}_{D,\text{nc}} = \pi_{\text{co}}$$

3/ Instrumental Variables

Identification Assumptions



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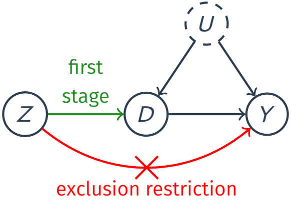

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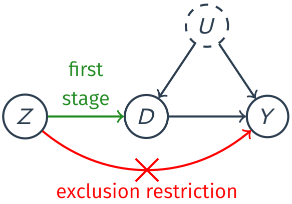

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  - Assignment to canvassing only affects turnout through actual canvassing
  - Not a testable assumption and cannot be guaranteed by design!
- Implies that potential outcomes only a function of D_i:

$$Y_i(1) = Y_i(D_i = 1) = Y_i(Z_i = 1, D_i = 1)$$

$$Y_i(0) = Y_i(D_i = 0) = Y_i(Z_i = 1, D_i = 0)$$

Outcome ITTs and Compliance Types

- We can define the ITTs on the outcome by compliance type as well.
  - $\text{ITT}_{Y,\text{co}} \rightsquigarrow$ effect of assignment on outcome among compliers.
  - $\text{ITT}_{Y,\text{nc}} \rightsquigarrow$ effect of assignment on outcome among noncompliers.
- Exclusion restriction has implications for these:
  - Implies that $\text{ITT}_{Y,\text{nc}} = 0$: if D_i does not change, Y_i cannot change.
  - Implies that $\text{ITT}_{Y,\text{co}}$ is due entirely to treatment uptake.

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  - Implies that $\text{ITT}_{Y,\text{co}}$ is due entirely to treatment uptake.
- Allows us to connect the ITT_Y on the outcome to compliance groups:

$$\begin{aligned}\text{ITT}_Y &= \pi_{\text{co}} \cdot \text{ITT}_{Y,\text{co}} + \pi_{\text{nc}} \cdot \text{ITT}_{Y,\text{nc}} \\ &= \text{ITT}_D \cdot \text{ITT}_{Y,\text{co}}\end{aligned}$$

LATE

- Under the exclusion restriction, $\text{ITT}_{Y,\text{co}}$ is the effect of treatment receipt/uptake:

$$\begin{aligned}\text{ITT}_{Y,\text{co}} &= \frac{1}{n_{\text{co}}} \sum_{i:C_i=\text{co}} Y_i(1, D_i(1)) - Y_i(0, D_i(0)) \\ &= \frac{1}{n_{\text{co}}} \sum_{i:C_i=\text{co}} Y_i(D_i = 1) - Y_i(D_i = 0) = \tau_{\text{LATE}}\end{aligned}$$

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    - It's a conditional ATE, where we condition on units being a complier.
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- This quantity is known as the **local ATE** (LATE), local to compliers.
  - It's a conditional ATE, where we condition on units being a complier.
  - Also referred to as **complier average causal effect** (CACE).
- **LATE theorem** under one-sided noncompliance, exclusion restriction, first-stage, and randomization:

$$\tau_{\text{LATE}} = \text{ITT}_{Y,\text{co}} = \frac{\text{ITT}_Y}{\text{ITT}_D}$$

Wald Estimator

- **Wald** or **instrumental variable estimator** for the LATE:

$$\widehat{\tau}_{\text{iv}} = \frac{\widehat{\text{ITT}}_Y}{\widehat{\text{ITT}}_D}$$

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  - Not 'unbiased', but it is **consistent** for $\tau_{\textbf{LATE}}$.

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- Equivalent to the **two-stage least squares** estimator:
  - Regress D_i on Z_i to get fitted values, $\widehat{D}_i$.
  - Regress Y_i on $\widehat{D}_i$.

- We can use the delta method to find the (superpopulation) variance:

$$\mathbb{V}[\widehat{\tau}_{\text{iv}}] = \frac{1}{\text{ITT}_D^2}\mathbb{V}\left[\widehat{\text{ITT}}_Y\right] + \frac{\text{ITT}_Y^2}{\text{ITT}_D^4}\mathbb{V}\left[\widehat{\text{ITT}}_D\right] - 2 \cdot \frac{\text{ITT}_Y}{\text{ITT}_D^3} \cdot \text{cov}\left[\widehat{\text{ITT}}_Y, \widehat{\text{ITT}}_D\right]$$

4/ Two-Sided Noncompliance

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    - Some may refuse encouragement and opt not to take treatment.
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- Z_i is the encouragement, and D_i is the treatment.
- No change in estimation, just different identification assumptions.

Compliance Types

- Four compliance types (or **principal strata**) in this setting:
  - Complier: $D_i(1) = 1$ and $D_i(0) = 0$
  - Always-taker: $D_i(1) = D_i(0) = 1$
  - Never-taker: $D_i(1) = D_i(0) = 0$
  - Defier: $D_i(1) = 0$ and $D_i(0) = 1$

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- Connections between observed data and compliance types:

	$Z_i = 0$	$Z_i = 1$
$D_i = 0$	Never-taker or Complier	Never-taker or Defier
$D_i = 1$	Always-taker or Defier	Always-taker or Complier

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- Let π_{co}, π_{at}, π_{nt}, and π_{df} be the proportions of each compliance type.
- ITT effects on D_i are more murky: $\text{ITT}_D = \pi_{\text{co}} - \pi_{\text{df}}$
  - Defiers really makes things messy!

Instrumental Variables Assumptions

- Canonical IV assumptions for Z_i to be a valid instrument:
  1. Randomization of Z_i
  2. Presence of some compliers $\pi_{\text{co}} \neq 0$ (first-stage)
  3. Exclusion restriction $Y_i(z, d) = Y_i(z', d)$
  4. **Monotonicity**: $D_i(1) \geq D_i(0)$ for all i (no defiers)

- Implies ITT effect on treatment equals proportion of compliers: $\text{ITT}_D = \pi_{\text{co}}$
- Implies that ITT for the outcome has the same interpretation:

$$\begin{aligned}\text{ITT}_Y &= \pi_{\text{co}} \cdot \text{ITT}_{Y,\text{co}} + \pi_{\text{at}} \cdot \underbrace{\text{ITT}_{Y,\text{at}}}_{=0 \text{ (ER)}} + \pi_{\text{nt}} \cdot \underbrace{\text{ITT}_{Y,\text{nt}}}_{=0 \text{ (ER)}} + \text{ITT}_{Y,\text{df}} \cdot \underbrace{\pi_{\text{df}}}_{=0 \text{ (mono)}} \\ &= \text{ITT}_{\text{co}} \pi_{\text{co}}\end{aligned}$$

- $\rightsquigarrow$ Same identification result: $\tau_{\text{LATE}} = \text{ITT}_Y / \text{ITT}_D$

Is the LATE Useful?

- The LATE is an effect on an unknown subset of the data.
    - Treated units are a mix of always takers and compliers.
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- Without further assumptions, $\tau_{\text{LATE}} \neq \tau$.

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    - Control units are a mix of never takers and compliers.
- Without further assumptions, $\tau_{\text{LATE}} \neq \tau$.
- Complier group depends on the instrument $\rightsquigarrow$ different IVs will lead to different identified estimands.
    - But, we cannot do any better in terms of point estimation without more assumptions.

In R: Wald and TSLS Estimator

```
# *Recall what we did in Neyman's approach*
my_data # data includes Z, D, and Y

# Proportion of compliers (using ITT_D)
pi_co <- mean(my_data$D[my_data$Z == 1]) - mean(my_data$D[my_data$Z == 0])

# Compute ITT's
ITT_Y <- mean(my_data$Y[my_data$Z == 1]) - mean(my_data$Y[my_data$Z == 0])
ITT_D <- mean(my_data$D[my_data$Z == 1]) - mean(my_data$D[my_data$Z == 0])
# (ITT_D = pi_co)

# TODO 1: Compute Wald estimator
Wald_est <- NULL

# TODO 2: Compute variance
# TODO 2-1: Compute variance terms using neyman estimator
Var_ITT_Y_est <- NULL
Var_ITT_Y_est <- NULL

# Compute covariance term
# demean
demeaned_y <- my_data$Y[my_data$Z == 1] - mean(my_data$Y[my_data$Z == 1])
demeaned_d <- my_data$D[my_data$Z == 1] - mean(my_data$D[my_data$Z == 1])
# denominator
denom <- sum(my_data$Z)*(sum(my_data$Z) - 1)
Covar_est <- (demeaned_y %*% demeaned_t)/denom

# TODO 2-2: Compute the estimate of the formula in p.6
Var_Wald_est <- NULL

# Or use TSLS with AER::ivreg
ivmodel <- AER::ivreg(Y ~ D | Z, data = my_data)
ivpack::robust.se(ivmodel)
```

Onto the presentations & discussions!

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